DOUBLE REWEIGHTED SPARSE REGRESSION FOR HYPERSPECTRAL UNMIXING

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ABSTRACT

Spectral unmixing is an important technology in hyperspectral image application. Recently, sparse regression is widely used in hyperspectral unmixing. This paper proposes a double reweighted sparse regression method for hyperspectral unmixing. The proposed method enhances the sparsity of abundance fraction in both spectral and spatial domains through double weights, in which one is used to enhance the sparsity of endmembers in the spectral library, the other is used to improve the sparseness of abundance fraction of every material. Experimental results on both synthetic and real hyperspectral data sets demonstrate effectiveness of the proposed method both visually and quantitatively.

Index Terms— Hyperspectral unmixing, sparse regression, double weights

1. INTRODUCTION

Due to the low spatial solution of the sensor, mixed pixels are often encountered in hyperspectral imagery (HSI). To some extent, the existence of mixed pixels will restrict the exploitation, processing, and applications of HSI in practice. Thus, spectral unmixing is an important technique for hyperspectral data exploitation, which decomposes a mixed pixel into a collection of constituent materials (also called endmembers) and their relative proportions (also called abundances) [1].

In the past few years, many methods have been proposed for hyperspctral unmixing. For the linear mixing model (LMM), it is assume that the spectrum of each pixel can be approximately represented by a linear mixture of endmember spectra weighted by the corresponding fractional abundances [2]. The geometrical and statistical frameworks [2] are two of widely used methods for hyperspectral unmixing. However, they generally require the presence of pure materials and the estimation of the number of endmembers in a given scene. Sparse unmixing [3], as semisupervised approach for linear spectral unmixing, has been approached in recent years. In sparse unmixing method, we try to find the optimal subset of signatures in a (potentially very large) spectral library that can best model each mixed pixel in the scene. Although sparse unmixing techniques have been shown obviously benefits, it is limited by the high correlation of spectral libraries. To improve the performance of sparse unmixing, spatial information of the given sence is taken into account. In [5], *sparse unmixing via variable splitting augmented Lagrangian and total variation* (SUnSAL-TV) is developed to sidestep the limitations of sparse unmixing. However, the sparsity in the HSI is not fully exploited and the estimated abundances by SUnSAL-TV are oversmooth.

In this paper, we propose a new sparse method for hyperspectral unmixing. The proposed method exploits double weights to enhance the sparsity of the abundance in both spectral and spatial domains. Extensive experiments on both synthetic and real hyperspectral data were conducted, whose results show the advantages of the proposed algorithm over some other hyperspectral unminxing approaches. The rest of this paper is organized as follows: Section 2 briefly describes the related work including linear sparse unmixing model and weighted l_1 minimization. Section 3 introduces the proposed method in detail. Our experimental results with simulated and real hyperspectral data sets is described in section 4. Section 5 concluds this paper.

2. RELATED WORK

2.1. Linear Sparse Unmixing Model

Linear sparse unmixing model [3] assumes that the observation of a mixed pixel can be expressed as a linear combination of available spectral signatures in a given spectral library $\mathbf{A} \in \mathbb{R}^{L\times m}$, i.e.,

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^{L\times 1}$ denotes a spectrum vector of a mixed pixel with L bands, $\mathbf{x}$ is an $m \times 1$ fractional abundance vector compatible with library $\mathbf{A} \in \mathbb{R}^{L\times m}$, and $\mathbf{n}$ is an $L \times 1$ vector collecting the errors affecting the measurements at each spectral band. The fractional abundance $\mathbf{x}$ generally satisfies the abundance nonnegativity constraint (ANC): $\mathbf{x} \geq 0$ and the abundance sum-to-one constraint (ASC): $\mathbf{1}^T\mathbf{x} = 1$. Due to

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the fact that only a few of the signatures in $\mathbf{A}$ likely contribute to the observed spectrum, the unmixing problem can be formulated as

$$\underset{\mathbf{x}}{min}\, \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2+\lambda\|\mathbf{x}\|_0, \qquad \text{s. t.} \qquad \mathbf{x}\geq 0 \tag{2}$$

2.2. Weighted l_1 Minimization

Sparse regression optimization can be represented as following:

$$\underset{\mathbf{x}}{min}\, \|\mathbf{x}\|_0, \quad \text{s. t. } \mathbf{A}\mathbf{x}=\mathbf{b}. \tag{3}$$

Its relaxing problem is

$$\underset{\mathbf{x}}{min}\, \|\mathbf{x}\|_1, \quad \text{s. t. } \mathbf{A}\mathbf{x}=\mathbf{b} \tag{4}$$

where $\mathbf{x}$ and $\mathbf{b}$ are $p\times 1$ and $L\times 1$ vectors. In order to enhance the sparsity of l_1 norm in (4), the approach of [6] proposed a weighted formulation of l_1 minimization,

$$\underset{\mathbf{x}}{min}\, \|\mathbf{W}\mathbf{x}\|_1, \quad \text{s. t. } \mathbf{A}\mathbf{x}=\mathbf{b} \tag{5}$$

where $\mathbf{W}=diag(w_1,w_2,...,w_p)$ is a diagonal matrix, whose entry $w_i^{t+1}=\frac{1}{|\mathbf{x}_i^t|+\varepsilon}$, with ε being a small positive vaule. This formulation suggests more generally that large weights could be used to discourage nonzero entries in the recovered signal, while small weights could be used to encourage nonzero entries.

3. METHODOLOGY

Inspired by the success of weighted l_1 minimization [6] in sparsity promotion, in this paper we propose a novel hyperspectral unmixing (HSU) method based on double reweighted sparse regression. As the weight is composed of two parts, the proposed method is called *double reweighted sparse unmixing* (DRSU).

To enhance the sparsity of solution and improve the unmixing quality, we propose the double reweighted sparse regression for hyperspectral unmixing as follows

$$\begin{aligned}&\underset{\mathbf{X}}{min}\, \frac{1}{2}\|\mathbf{Y}-\mathbf{A}\mathbf{X}\|_F^2+\lambda\|\mathbf{W_2}\cdot(\mathbf{W_1}\mathbf{X})\|_{1,1},\\ &\quad \text{s. t.} \qquad \mathbf{X}\geq 0\end{aligned} \tag{6}$$

where the operator $\cdot$ denotes the component-wise product (Hadamard product) of two variables. $\mathbf{W}_1\in\mathbb{R}^{p\times p}$ is a diagonal matrix, its diagonal elements can be computed as

$$\begin{aligned}&\mathbf{W}_1^{t+1}=diag(\frac{1}{\|\mathbf{X}^t(1,:)\|_2+\varepsilon},\frac{1}{\|\mathbf{X}^t(2,:)\|_2+\varepsilon},...,\\ &\frac{1}{\|\mathbf{X}^t(p,:)\|_2+\varepsilon})\end{aligned} \tag{7}$$

where $\mathbf{X}^t(i,:)$ is the ith row in the estimated abundance of the tth iteration. The weight $\mathbf{W}_1$ works similarly as [6] to promote the sparsity of the rows in the abundance matrix largely. However, the abundance distribution of each material is almost sparse in the scene. In the other words, the entries in an abundance image corresponding to each material should be sparse. $\mathbf{W}_1$ enhances the sparsity of spectra in library, but treats the entries in the same abundance image with same weight. Therefore, we propose the other weight $\mathbf{W}_2$ to further enhance the sparsity of the abundance matrix as:

$$\mathbf{W}_2^{l+1}=\frac{1}{\mathbf{X}_{i,j}^l+\varepsilon} \tag{8}$$

where $\mathbf{W}_2\in\mathbb{R}^{p\times N}$, $\mathbf{X}_{i,j}^t$ is the entry in the estimated abundance of the tth iteration. The large weights of $\mathbf{W}_2$ discourage nonzero entries in the estimated abundance, while small weights could be used to encourage nonzero entries. In DRSU, $\mathbf{W}_1$ enhances the sparsity of nonzero rows corresponding to the true endmembers in estimated abundance, while the sparsity of the nonzero entries in the nonzero rows is promoted by $\mathbf{W}_2$.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this section, unmixing experiments are performed on the synthetic and real hyperspectral images to illustrate the effectiveness of our proposed DRSU algorithm. Results of some very recent methods such as *sparse unmixing via variable splitting augmented Lagrangian* (SUnSAL) [3] and SUnSAL-TV [5] are given for comparison purpose.

4.1. Synthetic Data Experiments

The spectral library used in these synthetic image experiments is $\mathbf{A}\in\mathbb{R}^{224\times 240}$, which is generated by randomly selecting 240 different materials from the USGS library, available online at http://speclab.cr.usgs.gov/spectral.lib06. It comprises spectral signatures with reflectance values given in 224 spectral bands and distributed uniformly over the interval $0.4-2.5\mu$m. We simulate the synthetic data cube with 75×75 pixels and 224 bands per pixel based on LMM by using five randomly chosen spectral signatures from the library $\mathbf{A}$ as the endmembers and generating the abundances following the methodology of [5]. Finally, the simulated hyperspectral data is degraded by Gaussian noise with three levels of the signal-to-noise ratio, i.e., 20dB, 30dB, and 40dB.

We adopt the signal-to-reconstruction error (SRE) and the probability of success (p_s) as the objective metrics for quantitative evaluation. Specifically, the SRE in dB is defined as SRE $=10\log_{10}(E[\|\mathbf{x}\|_2^2]/E[\|\mathbf{x}-\hat{\mathbf{x}}\|_2^2])$ [3], and p_s is given by $p_s=P(\|\hat{\mathbf{x}}-\mathbf{x}\|^2/\|\mathbf{x}\|^2<3.16)$, where $\hat{\mathbf{x}}$ is the estimated fractional abundance vector of the true fractional abundance vector $\mathbf{x}$ and $E[\cdot]$ stands for mean value [3]. These metrics indicate the quality of the reconstruction of spectral mixtures.

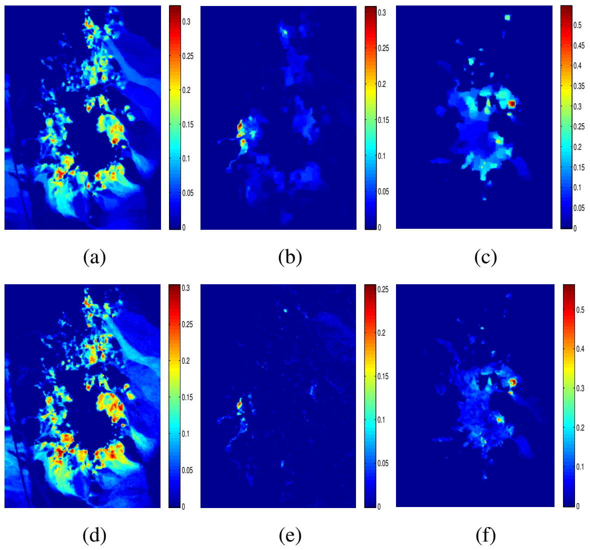

Fig. 1. Estimated abundance fractions for the Cuprite data: (from top to bottom) SUnSAL-TV and the proposed method.

The larger these values are, the better the performance of the algorithm for recovering the abundances are.

Table 1. Performance of different unmixing approaches on simulated hyperspectral data

SNR(in dB)	Algorithm	SRE(in dB)	p_s
20	SUnSAL	4.4661	0.3552
	SUnSAL-TV	10.8890	0.9889
	DRSU	**10.9537**	**1**
30	SUnSAL	9.4353	0.9292
	SUnSAL-TV	18.7212	1
	DRSU	**20.5721**	**1**
40	SUnSAL	11.1475	1
	SUnSAL-TV	28.1640	1
	DRSU	**30.6724**	**1**

For the latter, the real HSI used in the experiments is a subimage of 250×191 pixels and 188 bands from the publicly available AVIRIS Cuprite data collected in 1997. The Cuprite site is well understood mineralogically, and it has several exposed minerals of interest. The standard spectral library for this data is the USGS library containing 498 pure endmember signatures. Essential calibration was undertaken in order to mitigate the mismatches between the hyperspectral image and the signatures in the library [3]. The estimated results are demonstrated in Fig 1.

From Table 1 and Fig. 1, we can see that the abundance images estimated by SUnSAL-TV are oversmooth and the results of our method are more sparsity than SUnSAL-TV. On the whole, the proposed method outperforms the state-of-the-art hyperspectral unmixing methods.

5. CONCLUSIONS

To improve the accuracy of hyperspectral sparse unmixing, this paper proposes a double reweighted sparse unmixing method. The double weights improve the sparsity of the endmembers in spectral library and abundance fraction of every endmember. Simulated and real hyperspectral data sets are used to test the performance of the proposed DRSU. The experimental results in this paper consistently show that the DRSU method performs better than SUnSAL-TV. The future work will focus on choosing the parameters adaptively, as they affect the performance of unmixing significantly.

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